

## In The Claims

What is claimed is:

Claim 1 (canceled)

Claim 2 (currently amended): The system estimation method according to claim 17 ~~or 9 or 11~~, wherein the processing section calculates the existence condition in accordance with a following expression:

$$\hat{\Sigma}_{i|i}^{-1} = \hat{\Sigma}_{i|i-1}^{-1} + \frac{1 - \gamma_f^{-2}}{\rho} \mathbf{H}_i^T \mathbf{H}_i > 0, \quad i = 0, \dots, k \quad (17)$$

Claim 3 (currently amended): The system estimation method according to claim 17, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\varrho \hat{\Xi}_i + \rho \gamma_f^2 > 0, \quad i = 0, \dots, k \quad (18)$$

here,

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho \mathbf{H}_i \mathbf{K}_{s,i}}{1 - \mathbf{H}_i \mathbf{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \quad (19)$$

where the forgetting factor  $\rho$  and the upper limit value  $\gamma_f$  have a following relation:

$0 < \rho = 1 - \chi(\gamma_f) \leq 1$ , where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1) = 1$  and  $\chi(\infty) = 0$ .

Claims 4-6 (canceled)

Claim 7 (currently amended): ~~The system estimation method according to claim 1, wherein at the step of executing the hyper  $H_\infty$  filter,~~

~~the processing section calculates the~~ A system estimation method for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

$$\underline{x_{k+1}} = \underline{F_k x_k} + \underline{G_k w_k}$$

$$\underline{y_k} = \underline{H_k x_k} + \underline{v_k}$$

$$\underline{z_k} = \underline{H_k x_k}$$

here,

$x_k$ : a state vector or simply a state,

$w_k$ : a system noise,

$v_k$ : an observation noise,

$y_k$ : an observation signal,

$z_k$ : an output signal,

$F_k$ : dynamics of a system, and

$G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation method comprises:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ;

a step of executing a hyper  $H_\infty$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k$  and by following expressions (20) to (22), or, expression (20) and expressions which are deleted  $J_1^{-1}$  and  $J_1$  in the expressions (21) and (22),:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (20)$$

$$K_{s,k} = K_k(:,1)/R_{e,k}(1,1), \quad K_k = \rho^{\frac{1}{2}}(\rho^{-\frac{1}{2}}K_k R_{e,k}^{-\frac{1}{2}} J_1^{-1}) J_1 R_{e,k}^{\frac{1}{2}} \quad (21)$$

$$\begin{bmatrix} R_k^{\frac{1}{2}} & C_k \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ 0 & \rho^{-\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \end{bmatrix} \Theta(k) = \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & 0 \\ \rho^{-\frac{1}{2}} K_k R_{e,k}^{-\frac{1}{2}} J_1^{-1} & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}} \end{bmatrix} \quad (22)$$

Where,

$$\begin{aligned} R_k &= R_k^{\frac{1}{2}} J_1 R_k^{\frac{T}{2}}, \quad R_k^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}} \gamma_f \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{T}{2}} \\ R_{e,k} &= R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_1 R_{e,k}^{\frac{T}{2}}, \quad \hat{x}_{0|0} = \hat{x}_0 \end{aligned} \quad (23)$$

$\Theta(k)$  denotes a J-unitary matrix, that is, satisfies

$\Theta(k) J \Theta(k)^T = J$ ,  $J = (J_1 \oplus I)$ ,  $I$  denotes a unit matrix,

$K_k(:,1)$  denotes a column vector of a first column of the matrix  $K_k$ .

~~wherein  $J_1^{-1}$  and  $J_1$  can be deleted in the expressions (21) and (22),~~

here,

$\hat{x}_{k|k}$ : the estimated value of the state  $x_k$  at the time  $k$  using the observation signals  $y_0$  to  $y_k$ ,

$y_k$ : the observation signal,

$F_k$ : the dynamics of the system,

$K_{s,k}$ : the filter gain,

$H_k$ : the observation matrix,

$\hat{\Sigma}_{k|k}$ : corresponding to a covariance matrix of an error of  $\hat{x}_{k|k}$ ,  
 $\Theta(k)$ : the J-unitary matrix, and  
 $R_{e,k}$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.

Claim 8 (currently amended: The system estimation method according to claim 7, wherein the step of executing the hyper  $H_\infty$  filter includes:

a step at which the processing section calculates  $K_k$  and  $\hat{\Sigma}_{k+1|k}^{1/2}$  by using the expression (22);

a step at which the processing section calculates the filter gain  $K_{s,k}$  based on the an initial condition and of  $\hat{\Sigma}_{k|k-1}$  and an initial condition of  $C_k$ , and the matrix gain  $K_k$  by using the expression (21);

a step at which the processing section updates a filter equation of the  $H_\infty$  filter of the expression (20); and

a step at which the processing section repeatedly executes the respective steps the step of calculating by using the expression (20), the step of calculating by using the expression (21) and, the step of updating while advancing the time  $k$ .

Claim 9 (currently amended): ~~The system estimation method according to claim 1, wherein at the step of executing the hyper  $H_\infty$ -filter,~~

~~the processing section~~ A system estimation method for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which  
for a state space model expressed by following expressions:

$$\underline{x}_{k+1} = F_k \underline{x}_k + G_k w_k$$

$$\underline{y}_k = H_k \underline{x}_k + v_k$$

$$\underline{z}_k = H_k \underline{x}_k$$

here,

$\underline{x}_k$ : a state vector or simply a state,

$w_k$ : a system noise,

$v_k$ : an observation noise,

$\underline{y}_k$ : an observation signal,

$\underline{z}_k$ : an output signal,

$F_k$ : dynamics of a system, and

$G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation method comprises:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $\underline{y}_k$  as an input of a

filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ;

a step of executing a hyper  $H_\infty$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains the a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (61)$$

$$K_{s,k} = K_k(:, 1)/R_{e,k}(1, 1), \quad K_k = \rho^{\frac{1}{2}}(\bar{K}_k R_{e,k}^{-\frac{1}{2}}) R_{e,k}^{\frac{1}{2}} \quad (62)$$

$$\begin{bmatrix} R_{e,k+1}^{\frac{1}{2}} & 0 \\ \left[ \begin{array}{c} \bar{K}_{k+1} \\ 0 \end{array} \right] R_{e,k+1}^{-\frac{T}{2}} J_1 & \bar{L}_{k+1} R_{r,k+1}^{-\frac{T}{2}} \end{bmatrix} = \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & \check{C}_{k+1} \bar{L}_k R_{r,k}^{-\frac{1}{2}} \\ \left[ \begin{array}{c} 0 \\ \bar{K}_k \end{array} \right] R_{e,k}^{-\frac{1}{2}} J_1 & \rho^{-\frac{1}{2}} \bar{L}_k R_{r,k}^{-\frac{1}{2}} \end{bmatrix} \Theta(k) \quad (63)$$

here,  $\Theta(k)$  denotes an arbitrary J-unitary matrix, and  $\check{C}_k = \check{C}_{k+1} \Psi$  is established, where

$$\begin{aligned} R_k &= R_k^{\frac{1}{2}} J_1 R_k^{\frac{T}{2}}, \quad R_k^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}} \gamma_f \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{T}{2}} \\ R_{e,k} &= R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_1 R_{e,k}^{\frac{T}{2}}, \quad \hat{x}_{0|0} = \hat{x}_0 \end{aligned} \quad (23)$$

here,

$\hat{x}_{k|k}$ : the estimated value of the state  $x_k$  at the time  $k$  using the observation signals  $y_0$  to  $y_k$ ,

$y_k$ : the observation signal,

$K_{s,k}$ : the filter gain,

$H_k$ : the observation matrix,

$\Theta(k)$ : the J-unitary matrix, and

$R_{e,k}$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and  
a step at which the processing section sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.

Claim 10 (currently amended): The system estimation method according to claim 9, wherein the step of executing the hyper  $H_\infty$  filter includes:

a step at which the processing section calculates  $K_k^-$  based on an initial condition of  $R_{e,k+1}$ ,  $R_{r,k+1}$  and  $L_{k+1}^-$  by using the expression (63);

a step at which the processing section calculates the filter gain  $K_{s,k}^- - K_{s,k}$  based on the initial condition and by using the expression (62);

a step at which the processing section updates a filter equation of the  $H_\infty$  filter of the expression (61); and

a step at which the processing section repeatedly executes the respective steps the step of calculating by using the expression (63), the step of calculating by using the expression (62), and, the step of updating while advancing the time  $k$ .

~~Claim 11 (currently amended): The system estimation method according to claim 1, wherein at the step of executing the hyper  $H_\infty$  filter,~~

~~the processing section~~ A system estimation method for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which for a state space model expressed by following expressions:

$$\underline{x}_{k+1} = F_k \underline{x}_k + G_k w_k$$

$$\underline{y}_k = H_k \underline{x}_k + v_k$$

$$\underline{z}_k = H_k \underline{x}_k$$

here,

$\underline{x}_k$ : a state vector or simply a state,

$w_k$ : a system noise,

$v_k$ : an observation noise,

$\underline{y}_k$ : an observation signal,

$\underline{z}_k$ : an output signal,

$F_k$ : dynamics of a system, and

$G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation method comprises:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $\underline{y}_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ;

a step of executing a hyper  $H_\infty$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage



section and obtains the a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k^-$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}} \bar{K}_k(:, 1) / R_{e,k}(1, 1) \quad (26)$$

$$\begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (30)$$

Where,

$$\check{C}_{k+1} = \begin{bmatrix} H_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \text{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1) \times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \bar{K}_0 = 0, \quad \hat{x}_{0|0} = \hat{x}_0, \quad \bar{K}_k = \rho^{-\frac{1}{2}} K_k \quad (31)$$

wherein the above expressions can be arranged also with respect to  $K_k$  instead of  $K_k^-$

here,

$y_k$ : the observation signal,

$F_k$ : the dynamics of the system,

$H_k$ : the observation matrix,

$\hat{x}_{k|k}$ : the estimated value of the state  $x_k$  at the time  $k$  using the observation signals  $y_0$  to  $y_k$ ,

$K_{s,k}$ : the filter gain, obtained from the gain matrix  $K_k^-$ , and

$R_{e,k}$ ,  $\tilde{L}_k$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.

Claim 12 (canceled)

Claim 13 (currently amended): The system estimation method according to claim 17, wherein an estimated value  $z_{k|k}^v$  of the output signal is obtained from the state estimated value  $\hat{x}_{k|k}$  at the time  $k$  by a following expression:

$$z_{k|k}^v = H_k \hat{x}_{k|k}.$$

Claim 14 (currently amended): The system estimation method according to claim 17, wherein the  $H_\infty$  filter equation is applied to obtain the state estimated value  $\hat{x}_{k|k} = [\hat{h}_1[k], \dots, \hat{h}_N[k]]$

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{k-i}, \quad k = 0, 1, 2, \dots \quad (34)$$

and

an echo canceller is realized by canceling an actual echo by the obtained pseudo-echo.

Claim 15 (currently amended): A system estimation program for causing a computer to make state estimation robust and to

optimize a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

$$\underline{x}_{k+1} = F_k \underline{x}_k + G_k w_k$$

$$\underline{y}_k = H_k \underline{x}_k + v_k$$

$$\underline{z}_k = H_k \underline{x}_k$$

here,

$\underline{x}_k$ : a state vector or simply a state,

$w_k$ : a system noise,

$v_k$ : an observation noise,

$y_k$ : an observation signal,

$z_k$ : an output signal,

$F_k$ : dynamics of a system, and

$G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

~~for a state space model expressed by following expressions:~~

$$\underline{x}_{k+1} = F_k \underline{x}_k + G_k w_k$$

$$\underline{y}_k = H_k \underline{x}_k + v_k$$

$$\underline{z}_k = H_k \underline{x}_k$$

here,

~~$\underline{x}_k$ : a state vector or simply a state,~~

~~$w_k$ : a system noise,~~

~~$v_k$ : an observation noise,~~

~~$y_k$ : an observation signal,~~

~~$z_k$ : an output signal,~~

~~$F_k$ : dynamics of a system, and~~

~~$G_k$ : a drive matrix,~~

~~—— a maximum energy gain to a filter error from a disturbance weighted with the forgetting coefficient  $\rho$  as an evaluation criterion is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and~~

the system estimation program causes the computer to execute:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ;

a step of executing a hyper  $H_\infty$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k^-$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}} \bar{K}_k(:,1)/R_{e,k}(1,1) \quad (26)$$

$$\begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (30)$$

Where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \text{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1) \times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \bar{K}_0 = 0, \quad \hat{x}_{0|0} = \hat{x}_0, \quad \bar{K}_k = \rho^{-1} K_k \quad (31)$$

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here,

$y_k$ : the observation signal,

$F_k$ : the dynamics of the system,

$H_k$ : the observation matrix,

$\hat{x}_{k|k}$ : the estimated value of the state  $x_k$  at the time  $k$  using the observation signals  $y_0$  to  $y_k$ ,

$K_{s,k}$ : the filter gain, obtained from the gain matrix  $K_k^-$ , and

$R_{e,k}$ ,  $\tilde{L}_k$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section sets the upper limit value to be small within a range where the existence

condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.

~~— a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;~~

~~— a step at which the processing section determines the forgetting coefficient  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ;~~

~~— a step at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and uses the forgetting coefficient  $\rho$  to execute a hyper  $H_\infty$  filter expressed by a following expression:~~

$$\hat{x}_{k|k} = F_{k-1} \hat{x}_{k-1|k-1} + K_{g,k} (y_k - H_k F_{k-1} \hat{x}_{k-1|k-1})$$

~~here,~~

~~$\hat{x}_{k|k}$ , an estimated value of a state  $x_k$  at a time  $k$  using observation signals  $y_0$  to  $y_k$ ;~~

~~$F_k$ , dynamics of the system, and~~

~~$K_{g,k}$ , a filter gain,~~

~~— a step at which the processing section stores an obtained value relating to the hyper  $H_\infty$  filter into the storage section;~~

~~— a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting coefficient  $\rho$  by the obtained observation matrix  $H_k$  or the observation matrix  $H_k$  and the filter gain  $K_{g,k}$ ; and~~

~~— a step at which the processing section sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into~~

~~the storage section by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.~~

Claim 16 (currently amended): A computer readable recording medium recording a system estimation program for causing a computer to make state estimation robust and to optimize a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

$$\underline{x_{k+1} = F_k x_k + G_k w_k}$$

$$\underline{y_k = H_k x_k + v_k}$$

$$\underline{z_k = H_k x_k}$$

here,

$x_k$ : a state vector or simply a state,

$w_k$ : a system noise,

$v_k$ : an observation noise,

$y_k$ : an observation signal,

$z_k$ : an output signal,

$F_k$ : dynamics of a system, and

$G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

~~for a state space model expressed by following expressions:~~

$$\underline{x_{k+1} = F_k x_k + G_k w_k}$$

$$\underline{y_k = H_k x_k + v_k}$$

$$\underline{z_k = H_k x_k}$$

here,

~~$x_k$ : a state vector or simply a state,~~

~~$w_k$ : a system noise,~~

~~$v_k$ : an observation noise,~~

~~$y_k$ : an observation signal,~~

~~$z_k$ : an output signal,~~

~~$F_k$ : dynamics of a system, and~~

~~$G_k$ : a drive matrix,~~

~~—— a maximum energy gain to a filter error from a disturbance weighted with the forgetting coefficient  $\rho$  as an evaluation criterion is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and~~

the computer readable recording medium recording the system estimation program causes the computer to execute:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ;

a step of executing a hyper  $H_\infty$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k$  and by following expressions:



$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}} \bar{K}_k(:, 1) / R_{e,k}(1, 1) \quad (26)$$

$$\begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \bar{L}_k R_{r,k}^{-1} \bar{L}_k^T \check{C}_{k+1}^T \quad (27)$$

$$\bar{L}_{k+1} = \rho^{-\frac{1}{2}} \bar{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \bar{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \bar{L}_k R_{r,k}^{-1} \bar{L}_k^T \check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \bar{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \bar{L}_k \quad (30)$$

Where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \text{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\bar{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1) \times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \bar{K}_0 = 0, \quad \hat{x}_{0|0} = \hat{x}_0, \quad \bar{K}_k = \rho^{-\frac{1}{2}} K_k \quad (31)$$

---

here,

$y_k$ : the observation signal,

$F_k$ : the dynamics of the system,

$H_k$ : the observation matrix,

$\hat{x}_{k|k}$ : the estimated value of the state  $x_k$  at the time  $k$  using the observation signals  $y_0$  to  $y_k$ ,

$K_{s,k}$ : the filter gain, obtained from the gain matrix  $K_k^-$ , and

$R_{e,k}$ ,  $\bar{L}_k$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section sets the upper limit value to be small within a range where the existence

condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value  $\gamma_E$  and repeating the step of executing the hyper  $H_\infty$  filter. — a step at which a processing section inputs the upper limit value  $\gamma_E$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section, — a step at which the processing section determines the forgetting coefficient  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_E$ , — a step at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and uses the forgetting coefficient  $\rho$  to execute a hyper  $H_\infty$  filter expressed by a following expression:

$$\hat{x}_{k|k} = F_{k-1} \hat{x}_{k-1|k-1} + K_{\theta,k} (y_k - H_k F_{k-1} \hat{x}_{k-1|k-1})$$

here,

$\hat{x}_{k|k}$ , an estimated value of a state  $x_k$  at a time  $k$  using observation signals  $y_0$  to  $y_k$ ,

$F_k$ : dynamics of the system, and

$K_{\theta,k}$ , a filter gain,

— a step at which the processing section stores an obtained value relating to the hyper  $H_\infty$  filter into the storage section,

— a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_E$  and the forgetting coefficient  $\rho$  by the obtained observation matrix  $H_k$  or the observation matrix  $H_k$  and the filter gain  $K_{\theta,k}$ , and

— a step at which the processing section sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into

~~the storage section by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.~~

Claim 17 (currently amended): A system estimation device for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

$$\underline{x_{k+1} = F_k x_k + G_k w_k}$$

$$\underline{y_k = H_k x_k + v_k}$$

$$\underline{z_k = H_k x_k}$$

here,

$x_k$ : a state vector or simply a state,

$w_k$ : a system noise,

$v_k$ : an observation noise,

$y_k$ : an observation signal,

$z_k$ : an output signal,

$F_k$ : dynamics of a system, and

$G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

~~for a state space model expressed by following expressions:~~

$$\underline{x_{k+1} = F_k x_k + G_k w_k}$$

$$\underline{y_k = H_k x_k + v_k}$$

$$\underline{z_k = H_k x_k}$$

here,

~~$x_k$ : a state vector or simply a state,~~

~~$w_k$ : a system noise,~~  
 ~~$v_k$ : an observation noise,~~  
 ~~$y_k$ : an observation signal,~~  
 ~~$z_k$ : an output signal,~~  
 ~~$F_k$ : dynamics of a system, and~~  
 ~~$G_k$ : a drive matrix,~~  
~~— a maximum energy gain to a filter error from a~~  
~~disturbance weighted with the forgetting coefficient  $\rho$  as an~~  
~~evaluation criterion is suppressed to be smaller than a term~~  
~~corresponding to a previously given upper limit value  $\gamma_E$ ,~~  
the system estimation device comprises:  
a processing section to execute the estimation algorithm;  
and  
a storage section to which reading and/or writing is  
performed by the processing section and which stores  
respective observed values, set values, and estimated values  
relevant to the state space model,  
further comprising:  
a means at which the processing section inputs the upper  
limit value  $\gamma_E$ , the observation signal  $y_k$  as an input of a  
filter and a value including an observation matrix  $H_k$  from the  
storage section or an input section;  
a means at which the processing section determines the  
forgetting factor  $\rho$  relevant to the state space model in  
accordance with the upper limit value  $\gamma_E$ ;  
a means of executing a hyper  $H_\infty$  filter at which the  
processing section reads out an initial value or a value  
including the observation matrix  $H_k$  at a time from the storage  
section and obtains a filter gain  $K_{s,k}$  by using the forgetting  
factor  $\rho$  and a gain matrix  $K_k^-$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}} \bar{K}_k(:, 1) / R_{e,k}(1, 1) \quad (26)$$

$$\begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (30)$$

where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \text{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1) \times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \bar{K}_0 = 0, \quad \hat{x}_{0|0} = \hat{x}_0, \quad \bar{K}_k = \rho^{-1} K_k \quad (31)$$

---

here,

$y_k$ : the observation signal,

$F_k$ : the dynamics of the system,

$H_k$ : the observation matrix,

$\hat{x}_{k|k}$ : the estimated value of the state  $x_k$  at the time  $k$  using the observation signals  $y_0$  to  $y_k$ ,

$K_{s,k}$ : the filter gain, obtained from the gain matrix  $K_k$ , and

$R_{e,k}$ ,  $\tilde{L}_k$ : an auxiliary variable.

a means at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a means at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a means at which the processing section sets the upper limit value to be small within a range where the existence

condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value  $\gamma_f$  and repeating the means of executing the hyper  $H_\infty$  filter.

—— the processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section,

—— the processing section determines the forgetting coefficient  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ,

—— the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and uses the forgetting coefficient  $\rho$  to execute a hyper  $H_\infty$  filter expressed by a following expression:

$$\hat{x}_{k|k} = F_{k-1} \hat{x}_{k-1|k-1} + K_{\theta,k} (y_k - H_k F_{k-1} \hat{x}_{k-1|k-1})$$

here,

$\hat{x}_{k|k}$ ; an estimated value of a state  $x_k$  at a time  $k$  using observation signals  $y_0$  to  $y_k$ ,

$F_k$ ; dynamics of the system, and

$K_{\theta,k}$ ; a filter gain,

—— the processing section stores an obtained value relating to the hyper  $H_\infty$  filter into the storage section,

—— the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting coefficient  $\rho$  by the obtained observation matrix  $H_k$  or the observation matrix  $H_k$  and the filter gain  $K_{\theta,k}$ , and

—— the processing part sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section by decreasing the upper limit value  $\gamma_f$  and repeating the step of executing the hyper  $H_\infty$  filter.

Claim 18 (new): The system estimation method according to claim 9, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\varrho \hat{\Xi}_i + \rho \gamma_f^2 > 0, \quad i=0, \dots, k \quad (18)$$

here,

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho \mathbf{H}_i \mathbf{K}_{s,i}}{1 - \mathbf{H}_i \mathbf{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \quad (19)$$

where the forgetting factor  $\rho$  and the upper limit value  $\gamma_f$  have a following relation:

$0 < \rho = 1 - \chi(\gamma_f) \leq 1$ , where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1) = 1$  and  $\chi(\infty) = 0$ .

Claim 19 (new): The system estimation method according to claim 9, wherein an estimated value  $z_{k|k}^v$  of the output signal is obtained from the state estimated value  $\hat{x}_{k|k}$  at the time  $k$  by a following expression:

$$z_{k|k}^v = \mathbf{H}_k \hat{x}_{k|k}.$$

Claim 20 (new): The system estimation method according to claim 9, wherein the  $H_\infty$  filter equation is applied to obtain the state estimated value  $\hat{x}_{k|k} = [\hat{h}_1[k], \dots, \hat{h}_N[k]]$

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{k-i}, \quad k = 0, 1, 2, \dots \quad (34)$$

and

an echo canceller is realized by canceling an actual echo by the obtained pseudo-echo.

Claim 21 (new): The system estimation method according to claim 11, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\varrho \hat{\Xi}_i + \rho \gamma_f^2 > 0, \quad i=0, \dots, k \quad (18)$$

here,

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho \mathbf{H}_i \mathbf{K}_{s,i}}{1 - \mathbf{H}_i \mathbf{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \quad (19)$$

where the forgetting factor  $\rho$  and the upper limit value  $\gamma_f$  have a following relation:

$0 < \rho = 1 - \chi(\gamma_f) \leq 1$ , where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1) = 1$  and  $\chi(\infty) = 0$ .

Claim 22 (new): The system estimation method according to claim 11, wherein an estimated value  $z_{k|k}^v$  of the output signal is obtained from the state estimated value  $\hat{x}_{k|k}$  at the time  $k$  by a following expression:

$$z_{k|k}^v = \mathbf{H}_k \hat{x}_{k|k}.$$

Claim 23 (new): The system estimation method according to claim 11, wherein the  $H_\infty$  filter equation is applied to obtain the state estimated value  $\hat{x}_{k|k} = [\hat{h}_1[k], \dots, \hat{h}_N[k]]$

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{k-i}, \quad k = 0, 1, 2, \dots \quad (34)$$

and

an echo canceller is realized by canceling an actual echo by the obtained pseudo-echo.